Construct a Demand Function Toward EOQ Model for Single-Period Products

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ABSTRACT

Due to the advance of technology, manufacturers' marketing strategies and quickly changing consumer tastes, numerous products such as fashionable apparel, clothing accessories and consumer electronics are liable to face the problem of short lifespan and obsolescence. Therefore, three related aspects are addressed in this paper. Firstly, we derive a time-dependent demand model for single-period products, well portraying an integrated demand behavior from growth and maturity stages to the final decay stage. Secondly, adding the factor of selling price, a novel time and selling price dependent-demand rate is accordingly presented. Finally, based on the demand, we explore an EOQ model for the purpose of total profit maximization, accompanied with numerical example illustration.
Keywords: *EOQ; Single-period; Inventory; Holding cost*

INTRODUCTION

One area of inventory management that has been extensively studied is the single-period model, where the demand of products occurs during a relatively short time and there exists only one opportunity to procure products at the beginning of the selling period. Meanwhile, products remaining in inventory at the end of the selling period will be discarded, salvaged, sold at a loss or entered as a tax write-off. In practice, retailers usually implement price policy to spur sales to avoid ending the period with excessive inventory.

Urban and Baker (1997) studied optimal ordering and pricing policies in a single-period environment with multivariate demand and markdowns, in which demand is a deterministic function of time, price and level of inventory. Gupta et al. (2006) developed a pricing model for clearing end-of-season retail inventory by computing optimal prices with demand that could be deterministic or stochastic, and arbitrarily correlated across planning period. In addition, You and Hsieh (2007) investigated a continuous inventory model to find strategies to sell a season item over a finite time interval. Dutta et al. (2005) proposed a single-period inventory model with fuzzy random variables.

Furthermore, Ghare and Schrader (1963) discussed a standard EOQ model to include exponential decay, where the lifetime of products is a random variable with negative exponential distribution. Delft and Vial (1996) also analyzed optimal policy for items with assumption of a negative exponential distribution to obsolescence. Arcelus et al. (2002) discussed inventory policy for items recognized as being subject to obsolescence, in which demand is assumed to be a decreasing function of both selling price and time, and up to a certain stochastic time point, suddenly drops to zero.

The purpose of this paper is to provide a framework for single-period product inventory policy, where we suppose it bears the following three features: (1) lifespan of products is relatively short; (2) beyond a certain time point, demand decreases with respect to time, but never reaches zero; (3) during the period of declining demand, the retailer offers price discounts to stimulate demand. Ultimately, mathematical models for demand and objective are formulated, and theoretical analysis is conducted to optimize the objective of total profit.

DERIVE THE DEMAND MODEL

In 1840, the Belgian mathematician Verhulst presented a model for population increase rate, saying that the population increase rate should be in direct linear proportion to the population amount at that time, and the total population amount would never exceed a certain maximum value restricted by their surrounding environment. He thereby offered an expression for the behavior of the population increase rate as following:

$$\frac{dP(t)}{dt} = \lambda P(t)\left(U - P(t)\right) \quad (1)$$

where $P(t)$ is the population amount at time t, λ is a positive constant and U represents maximal population amount.

As we determine that products are made for people to purchase and consume, thus we believe "the more people, the more demand". Furthermore, it is reasonable to adopt Verhulst's population increase rate model for product demand. In addition, to feature the characteristics of the products, we will extend his model to following two scenarios.

The first scenario time interval is $[0,\mu]$, and the demand rate $D_1(t)$ with the assumed upper bound U is increasing as time t increases, thus the variation of $D_1(t)$ with respect to t is obtained by replacing $P(t)$ with $D_1(t)$ in (1), and will be expressed in (2). Contrarily, the second scenario time interval is $t > \mu$, and the demand rate $D_2(t)$ with the certain lower bound "zero" is decreasing as time t increases. Hence, the variation of $D_2(t)$ with respect to time t is then obtained by replacing $D_1(t)$ with $D_2(t)$, U with zero in (2), and will be expressed in (3). Consequently, we have an integrated demand model for single-period products as follows.

$$\frac{dD_1(t)}{dt} = \lambda D_1(t)\left(U - D_1(t)\right), \quad 0 \le t \le \mu \quad (2) \text{ with } D_1(0) = D_0$$

$$\frac{dD_2(t)}{dt} = \lambda D_2(t)\left(0 - D_2(t)\right), \quad t > \mu \quad (3)$$

with $D_2(\mu) = D_1(\mu)$

where μ = time to peak demand

λ = positive constant

D_0 = initial demand

Solutions of (2) and (3) are given below and depicted in Fig. 1.

$$D_1(t) = \frac{U}{1 + ke^{-\lambda U t}} \quad , \quad 0 \le t \le \mu \quad (4)$$

$$D_2(t) = \frac{U}{\lambda U(t-\mu) + \delta} \quad , \quad t \ge \mu \quad (5)$$

where $k = \frac{U}{D_0} - 1, \qquad \delta = 1 + ke^{-\lambda U \mu}$

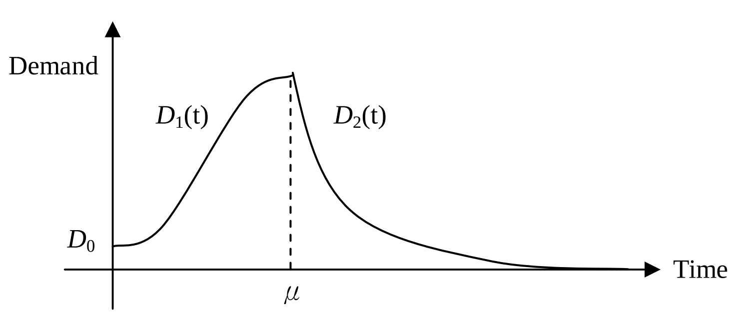

Figure 1: Demand without exogenous effects

ASSUMPTIONS AND NOTATION

The mathematical model proposed in this paper is based on the following assumptions.

An inventory management for a single-period product is explored, where the retailer initially sets a regular selling price p_0 and the original demand rate is characterized by (4) and (5). The retailer, however, intends to reset the selling price in order to promote sales when demand starts declining. As a result, the previously developed rates (4) and (5) are then modified by the following equation which is illustrated in Fig. 2. The retailer's objective is to determine the discount selling price *p* and order quantity so as to maximize his(her) total profit.

$$D(t,p)=\begin{cases} D_1(t)=\dfrac{U}{1+ke^{-\lambda Ut}} & ,0\le t\le \mu \\ D_2(t)+a(p_0-p)=\dfrac{U}{\lambda U(t-\mu)+\delta}+a(p_0-p), & t\ge \mu \end{cases} \tag{6}$$

where *a* is a positive constant.

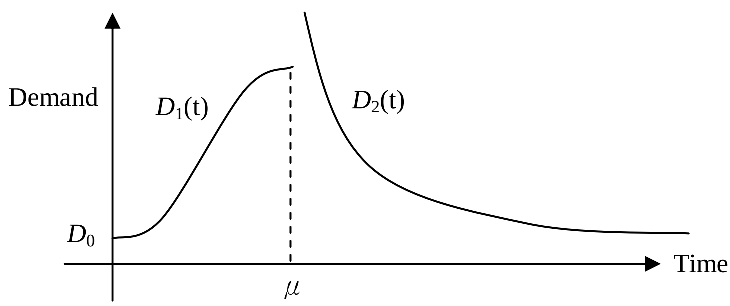

Figure 2: Demand with price consideration

In addition to the notation in Section 2, the following will be used throughout this paper.

$I(t)$ = inventory level at time *t*
q = order quantity (decision variable)
T = selling period
p_0 = regular selling price
P = discount selling price (decision variable)
A = ordering cost per order
c = unit purchasing cost
h = unit holding cost per unit time
K= cost incurred by price change, including changing of price lists, tags, product catalogues and advertising of price change, etc.

MODEL FORMULATION AND NUMERICAL EXAMPLE

According to the aforementioned assumptions, the instantaneous inventory level $I(t)$ is governed by the following differential equations.

$$\begin{cases} \dfrac{dI(t)}{dt}=-\dfrac{U}{1+ke^{-\lambda Ut}} & , \quad 0\le t\le \mu \\ \dfrac{dI(t)}{dt}=-\dfrac{U}{\lambda U(t-\mu)+\delta}-a(p_0-p), & \mu\le t\le T \end{cases} \tag{7}$$

with the initial condition $q=I(0)$ and boundary condition $I(T)=0$.

And the solutions of $I(t)$ and q are

$$I(t)=\begin{cases} q-\frac{1}{\lambda}\ln\frac{1+ke^{-\lambda Ut}}{(1+k)e^{-\lambda Ut}} &, \quad 0\le t\le\mu \\ \frac{1}{\lambda}\ln\frac{\lambda U(T-\mu)+\delta}{\lambda U(t-\mu)+\delta}+a(p_0-p)(T-t) &, \quad \mu\le t\le T \end{cases} \tag{8}$$

$$q=\frac{1}{\lambda}\ln\frac{(1+ke^{-\lambda U\mu})(\lambda U(T-\mu)+\delta)}{\delta(1+k)e^{-\lambda U\mu}}+a(p_0-p)(T-\mu) \tag{9}$$

Next, the retailer's total profit will be constructed, which consists of following five elements.

sale revenues = $p_0(q-I(\mu))+pI(\mu)=p_0q-(p_0-p)I(\mu)$

ordering cost = A

price changing cost = K

purchasing cost = $c\,q$

holding cost = $h\int_0^{\mu}I(t)dt+h\int_{\mu}^{T}I(t)dt$

$$=hq\mu-\frac{h}{\lambda}\left(\int_0^{\mu}\ln\frac{1+ke^{-\lambda Ut}}{(1+k)e^{-\lambda Ut}}dt-\int_{\mu}^{T}\ln\frac{\lambda U(T-\mu)+\delta}{\lambda U(t-\mu)+\delta}dt\right)$$
$$+\frac{1}{2}ah(p_0-p)(T-\mu)^2$$

Consequently, the total profit, denoted by TP, is calculated by

TP = *sale revenues- ordering cost- price changing cost- purchasing cost - holding cost*

$$=(p_0-c-h\mu)q-\frac{p_0-p}{\lambda}\ln\frac{\lambda U(T-\mu)+\delta}{\delta}-a(p_0-p)^2(T-\mu)$$
$$+\frac{h}{\lambda}\left(\int_0^{\mu}\ln\frac{1+ke^{-\lambda Ut}}{(1+k)e^{-\lambda Ut}}dt-\int_{\mu}^{T}\ln\frac{\lambda U(T-\mu)+\delta}{\lambda U(t-\mu)+\delta}dt\right)-\frac{1}{2}ah(p_0-p)(T-\mu)^2-A-K \tag{10}$$

Theorem 1. The TP is strictly concave in p.

Proof. First, taking the first-order derivative of q with respect to p, then $q'=-a(T-\mu)$ is obtained. Next, taking the first and second-order derivatives of TP with respect to p, we have

$$\text{TP}'=-a(p_0-c-h\mu)(T-\mu)+\frac{1}{\lambda}\ln\frac{\lambda U(T-\mu)+\delta}{\delta}$$
$$+2a(p_0-p)(T-\mu)+\frac{1}{2}ah(T-\mu)^2 \tag{11}$$

And TP" = $-2a(T-\mu)<0$ for all p, and this completes the proof.

Theorem 2. The optimal discount selling price and order quantity are uniquely determined by

$$p^*=p_0-\frac{1}{2}(p_0-c-h\mu)+\frac{1}{2\lambda a(T-\mu)}\ln\frac{\lambda U(T-\mu)+\delta}{\delta}+\frac{1}{4}h(T-\mu) \tag{12}$$

$$q^*=\frac{1}{\lambda}\ln\frac{(1+ke^{-\lambda U\mu})[\lambda U(T-\mu)+\delta]}{\delta(1+k)e^{-\lambda U\mu}}+\frac{1}{2}a(p_0-c-h\mu)(T-\mu)$$
$$-\frac{1}{2\lambda}\ln\frac{\lambda U(T-\mu)+\delta}{\delta}-\frac{1}{4}ah(T-\mu)^2 \tag{13}$$

Proof. According to the first-order necessary condition, P^* can be easily obtained by setting TP'=0 in (11). Moreover, by substituting the P^* value into (9), it immediately yields the result of (13).

Example. Parameters values are: U=1000, A=1000, λ =0.01,
$p_0 = 100, \mu = 2, T = 3, K = 200, h = 0.5, D_0 = 90, a = 50, c = 30.$

Then, from (12) and (13), we derive the optimal discount selling price $p^* = 68.02$ and the optimal order quantity $q^* = 3597.85$. Also, from (10), the maximal total profit TP=188600.

CONCLUSION

This paper dealt with the inventory policy in response to the single-period products, whose demand runs through the entire lifespan with the stage of growth, maturity and decay. To the best of our knowledge, we are the first to introduce the following two innovative proposals in the literature. First, without considering any possible exogenous factors, we derived a time-dependent demand model (2) and (3) for the products, which fits well into an integrated demand pattern. The second is that, according to the widely used formula $D = \alpha - \beta p$, we incorporated price effect into (3) by: the increase in demand due to price change is linearly correlated with the difference between the regular price and the discount price, and in subsequence a novel time and price-dependent demand model is accordingly presented to construct an EOQ model toward the products, aiming at maximizing the total profit.

For further study, the proposed EOQ model can be applied to products that suffer from deterioration. Also, we can generalize it to the models with multi-discount in price for either equal or unequal time subintervals. In short, this paper provides a concrete foundation toward single-period products that other researchers can follow in future.

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