The Investment under Uncertainty and the Preference for Capital

Dr. Young Seob Son, Associate Professor of Bemidji State University, USA

ABSTRACT

This paper investigates the investment behavior of a profit maximizing firm under uncertainty and the preference for capital. It shows that the intertemporal elasticity of substitution determines the sign of the investment-uncertainty relationship while the magnitude of risk aversion affects the scale of impact of uncertainty on investment. It also shows that the preference for capital may paradoxically decrease the rate of capital accumulation under uncertainty.

Keywords: Investment, Uncertainty, Risk aversion; Intertemporal elasticity of substitution, Preference for capital

INTRODUCTION

The relationship between investment and uncertainty has been an interesting and important topic in economics and finance. The role of the preference for capital has been analyzed in the area of economic growth with or without uncertainty. This paper introduces a way to analyze the investment behavior of a profit maximizing firm under uncertainty with the preference for capital (total assets).

This paper shows that the intertemporal elasticity of substitution determines the sign of the investment-uncertainty relationship while the magnitude of risk aversion affects the scale of impact of uncertainty on investment. It also shows that the preference for capital may paradoxically decrease the rate of capital accumulation under uncertainty even though the preference for capital increases the rate of capital accumulation if there is no uncertainty.

Nakamura (1999) demonstrated the negative relationship between investment and uncertainty for a competitive firm assuming that firms are risk-averse and the output price follows a Brownian motion. Unfortunately, his formation of the maximization problem did not have a closed form solution. Saltari and Ticchi (2005) proposed an alternative framework to have a closed form solution assuming that capital fully depreciates after production. Femminis (2008) showed that the investment-uncertainty relation is negative for values of the risk-aversion index larger than unity considering realistic values for the capital depreciation. Bhamra and Uppal (2006) found that consumption and portfolio decisions depend on both risk aversion and elasticity of intertemporal substitution. They showed that the size of risk aversion relative to unity determines the sign of the intertemporal hedging portfolio, while elasticity of intertemporal substitution affects only its magnitude. Saltari and Ticchi (2007) found risk aversion cannot by itself explain a negative relationship between aggregate investment and aggregate uncertainty, as the effect of increased uncertainty on investment also depends on the intertemporal elasticity of substitution. Chevalier-Roignant, Flath, Huchzermeier, and Trigeorgis (2011) provided an overview of the literature on strategic investment under uncertainty and stressed new insights from a selection of articles which deals with lumpy investment decisions, capital expansion models wherein it is recognized that firms may continuously adjust their capital stocks, and more complex investment problems in which several
investment decisions are intertwined. Chronopoulos, Reyck, and Siddiqui (2011) confirmed that risk aversion lowers the probability of investment and demonstrated how this effect can be mitigated by incorporating operational flexibility in the form of embedded suspension and resumption options. Kort, Murto, and Pawlina (2010) questioned how uncertainty in market development affects the trade-off: lumpy investment has a lower total cost, but stepwise investment gives more flexibility by letting the firm choose the training individually for each stage. The result was that higher uncertainty makes the single-stage investment more attractive relative to the more flexible stepwise investment strategy. Gulen and Ion (2016) found a strong negative relationship between firm-level capital investment and the aggregate level of uncertainty associated with future policy and regulatory outcomes using a news-based index of policy uncertainty and provided empirical support to the notion that policy uncertainty can depress corporate investment by inducing precautionary delays due to investment irreversibility. Baker, Bloom, and Davis (2016) developed a new index of economic policy uncertainty (EPU) based on newspaper coverage frequency. They found that policy uncertainty is associated with greater stock price volatility and reduced investment and employment in policy-sensitive sectors like defense, health care, finance, and infrastructure construction using firm-level data. In addition, at the macro level, they found innovations in policy uncertainty foreshadow declines in investment, output, and employment in the United States. Gilchrist, Sim, and Zakrajsek (2014) tried to answer the following question using two different channels: how much of the impact of fluctuations in uncertainty on aggregate investment reflects capital adjustment frictions associated with irreversibility—the traditional wait-and-see effect—and how much of it can be attributed to distortions in financial markets? They developed a quantitative business cycle model to analyze the effect of fluctuations in uncertainty on investment dynamics in a general equilibrium setting featuring partial irreversibility and frictions in both the debt and equity markets. Through model simulations, they found that financial frictions are a powerful conduit through which uncertainty shocks affect aggregate investment and capital liquidity shocks can be an important source of macroeconomic fluctuations. Glover and Levine (2015) showed that manager incentives should be considered when the relationship between uncertainty and investment is investigated. Huginnier, Malamud, and Erwan (2014) studied a dynamic model of cash management, financing, and investment decisions in which the Modigliani and Miller assumption of infinitely elastic supply of capital is relaxed and firms have to search for investors when in need of funds in order to find out when and how capital supply uncertainty affects real investment and determine the effects of capital market frictions on firms' financing and cash management policies. They found that firms with high investment costs differ in their behaviors from firms with low investment costs, financing policy does not follow a strict pecking order, and the optimal payout policy may feature several regions with both incremental and lumpy dividend payments. Wang, Wei, and Song (2017) studied the effects of policy and market uncertainties on corporate R&D investment using data from Chinese listed firms. They found that both policy and market uncertainties can negatively affect corporate R&D investment. Bloom (2014) casted four questions. First, what are some facts and patterns about economic uncertainty? Second, why does uncertainty vary during business cycles? Third, do fluctuations in uncertainty affect behavior? Fourth, has higher uncertainty worsened the Great Recession and slowed the recovery? He concluded the empirical literature on uncertainty and the literature on the policy implications of uncertainty are at an early stage even though the empirical progress on fluctuations in uncertainty over last decade has been exciting. Many policy questions also remain.

The contributions of this paper are adopting the financial ratios to identify the investment-uncertainty relationship and analyzing the role of the preference for capital (total assets) on investment.
The financial ratios are used in the ratio analysis in finance. The ratios that we used are total asset turnover, return on investment ratio, and payout ratio.

For simplicity, a representative firm is considered. A number of agents (shareholders) invest equally and take a proportion of earnings like a dividend. Total amount of investment comes from internal resources which are from net profit. The goal of this representative firm is to maximize the earnings (profits) and try to increase total assets ($k_t$) which can be considered as total wealth. In order to consider that the firm has the preference for capital, the utility function of this representative firm consists of two variables (earnings and capital).

The reminder of this study is presented as follows. Section 2 describes the basic model. Section 3 derives the equilibrium rate of net investment. Section 4 analyzes the effects of uncertainty and the preference for capital (total assets) on the equilibrium rate of net investment. Concluding remarks are found in the last section.

**THE MODEL**

**The Flow of Investment**

Suppose that there is a representative firm. The firm uses only one input, which is capital ($k_t$) and pays the dividend ($D_t$). Capital($k_0$) represents total assets which include both physical capital and human capital. The representative firm is endowed with the initial investment, ($k_0$), at time zero. Revenue ($R_t$) is $p_t y_t$, where $p_t$ and $y_t$ are the price and the production at time $t$, assuming our representative firm is a price taker and can sell as many as it can produce. The price ($p_t$) is normalized to one for simplicity. In other words, the revenue will be determined by the production ($y_t$) which depends on capital ($k_t$). The flow of revenue is assumed to follow the stochastic process:

$$dR_t = dy_t = a k_t (dt + \sigma dz_t) \quad (1)$$

where $\sigma$ is the instantaneous standard deviations of the innovations and $dz_t$ is the increment to a standard-normal Wiener process, $z_t$.

Finally, the flow of investment is

$$I_t = k_{t+1} - k_t = R_t - D_t - OE_t \quad (2)$$

where $I_t$ denotes the investment and $\delta$ denotes the depreciation rate which is constant and the operating expenses ($OE_t$) which are costs associated with running a business's core operations on a daily basis.

According to equation (2), investment hinges on capital accumulation, $k_{t+1} - k_t$, which is net investment. In order to identify the flow of net investment, we use the ratios, such as total asset turnover ratio, return on investment ratio, payout ratio, and operating expense ratio:

**Total asset turnover ratio**

$$\text{Total asset turnover ratio } (a) = \frac{\text{Revenue } (R_t)}{\text{Total assets } (k_t)} \quad (3)$$

**Return on investment ratio**

$$\text{Return on investment ratio } (\eta) = \frac{\text{Earnings } (\pi_t)}{\text{Total assets } (k_t)} \quad (4)$$

**Payout ratio**

$$\text{Payout ratio } (\mu) = \frac{\text{Dividends } (D_t)}{\text{Earnings } (\pi_t)} \quad (5)$$

**Operating expense ratio**

$$\text{Operating expense ratio } (o) = \frac{\text{Operating expenses } (OE_t)}{\text{Revenue } (R_t)} \quad (6)$$

The total asset turnover ratio indicates how effectively the firm uses its total resources. The return on investment ratio measures the rate of return on the total asset investment. The expense ratio indicates the proportion of revenue used for expenses. The total asset turnover ratio and the operating expense ratio are constant by assumption. If the dividend is considered as wage, it is reasonable for the representative firm to keep certain proportion ($\mu$) of profits as dividend. Now net investment ($k_{t+1} - k_t$) using the ratios is

$$k_{t+1} - k_t = R_t - D_t - OE_t - \delta k_t = (1 - o)ak_t - \mu k_t - \delta k_t = ((1 - o)a - \delta - \mu \eta)k_t \quad (7)$$

Net investment (capital accumulation) over time periods is as follows:

$$dk_t = ((1 - o)a - \delta - \mu \eta)k_t dt + a \sigma k_t dz_t \quad (8)$$
Preference

The objectives of this study are to analyze the effects of uncertainty and the preference for capital (total assets) on investments. In order to disentangle the effects of the elasticity of intertemporal substitution, risk aversion, and intra-period substitution between earnings ($\pi_t$) and capital ($k_t$), a class of preferences that is tractable, yet rich enough to capture these three distinct forces is adopted:

\[(1 - \gamma)U_t = \lim_{\Delta t \to 0} e^{-\rho \Delta t} \left\{ \left( \frac{\pi_t}{k_t} \right)^{\theta} \Delta t + e^{-\rho \Delta t} \left( (1 - \gamma)E_t U_{t+\Delta t} \right)^{\theta} \right\} \]  

(9)

This is a form of generalized isoelastic (GIE) preferences defined over the two “goods,” earnings ($\pi_t$) and capital ($k_t$). The aggregate function $X_t = \pi_t k_t^\theta$ is the intra-period felicity function. The rate of time preference is $\rho > 0$. Here, the parameter $\gamma$ is the coefficient of relative risk aversion for timeless lotteries of $X_t$, while $\epsilon = 1/(1-\theta)$ is the elasticity of intertemporal substitution for riskless paths of $X_t$. To analyze this model, the magnitudes of $\epsilon$ and $\gamma$ are crucial. According to most empirical estimates of $\gamma$ and $\epsilon$, there is a consensus that $\gamma > 1$ and there is a plausible case, $\epsilon > 1$, while the conventional wisdom is $\epsilon < 1$ (Smith and Son, 2005).

The Growth Rate of Investment under Uncertainty

The representative firm’s problem is to choose the optimal ratio of return on investment to maximize (9) subject to the flow constraint, (8), over an infinite planning horizon. The optimal ratio of return on investment is

\[ \eta = \frac{1}{(1+\mu)} \left\{ \frac{\alpha \rho}{\alpha+\beta} (1-\epsilon) \left[ \frac{\alpha \delta - (1-(\alpha+\beta)(1-\gamma)) a^2 \sigma^2}{\epsilon + (1-\epsilon) a} \right] \right\} \]  

(10)

It is clear that the preference for capital ($\beta$) affects the optimal ratio of return on investment by altering both risk aversion and the rate of time preference. To make this clear let us follow Smith (1999) in defining some new terms. Let the effective rate of time preference ($H$), the effective elasticity of intertemporal substitution ($E$), and the effective coefficient of relative risk aversion ($R$) be

\[ H = \frac{\alpha \rho}{\alpha+\beta} \]  

(11)

\[ E = \frac{\epsilon}{\epsilon + (1-\epsilon) a} \]  

(12)

\[ R = 1 - (\alpha + \beta)(1-\gamma) \]  

(13)

Given $\gamma > 1$, which all empirical estimates indicate, then an increase in the preference for capital ($\beta$) will increase the effective coefficient of relative risk aversion. However, an increase in $\beta$ unambiguously reduces the effective rate of time preference. Preferences for the resource stock have no effect on the effective elasticity of intertemporal substitution, $E$ With these terms, the optimal ratio of return on investment in Equation (10) can be rewritten as

\[ \eta = \frac{1}{1+\mu} \left\{ EH + (1-E) \left[ a - \delta - R \frac{a^2 \sigma^2}{2} \right] \right\} \]  

(14)

The preference for capital ($\beta$) affects $\pi_t$ by changing $H$ and $R$, effective time preference and risk aversion. Given that, the equilibrium rate of net investment is

\[ g = (1-o) a - \delta - \left\{ \frac{\epsilon \alpha \rho}{\alpha+\beta} + \alpha (1-\epsilon) \left[ a - \delta - (1-(\alpha+\beta)(1-\gamma)) \frac{a^2 \sigma^2}{2} \right] \right\} \frac{\epsilon}{\epsilon + (1-\epsilon) a} \]

\[ = (1-o) a - \delta - \left\{ EH + (1-E) \left[ a - \delta - R \frac{a^2 \sigma^2}{2} \right] \right\} \]

\[ = E \left( (1-o) a - \delta \right) - \left\{ EH - (1-E) \left[ R \frac{a^2 \sigma^2}{2} \right] \right\} \]  

(15)
Properties of the Growth Rate of Investment under Uncertainty

First consider the effect of uncertainty on the growth rate of investment.

\[
\frac{\partial g}{\partial \sigma^2} = (1 - E) \frac{\sigma^2 R}{\sigma^2} = \frac{\sigma^2(1-\epsilon)}{\epsilon(1-\epsilon)}(1 - (\alpha + \beta)(1 - \gamma)) \frac{\sigma^2}{\epsilon(1-\epsilon)} \tag{16}
\]

Given \( \gamma > 1 \), the sign of \( \frac{\partial g}{\partial \sigma^2} \) will depend on the magnitude of the elasticity of substitution. If \( \epsilon > 1 \) \((\epsilon > 1)\), the increase in uncertainty decreases (increases) investment. However, the magnitude of risk aversion and the preference for capital affect the scale of impact of uncertainty on the growth rate of investment.

In sum, given \( \gamma > 1 \), the intertemporal elasticity of substitution \( (\epsilon) \) determines the sign of \( \frac{\partial g}{\partial \sigma^2} \) and the degree of risk aversion \( (\gamma) \) and the preference for capital \( (\beta) \) affect the magnitude of \( \frac{\partial g}{\partial \sigma^2} \).

The Preference for Capital

Now consider the effect of the preference for capital.

\[
\frac{\partial g}{\partial \beta} = \frac{\alpha}{\epsilon(1-\epsilon)\alpha(\alpha + \beta)^2} - \frac{\epsilon \rho (1-\epsilon)(1-\gamma) \sigma^2}{\epsilon(1-\epsilon)\alpha} \tag{16}
\]

There are two effects of the preference for capital \( (\beta) \) on the growth rate of investment. The first effect is the “certainty effect” of the preference for capital \( (\beta) \), which is the first term in the equation (17). This effect increases investment through decreasing the effective rate of time preference \( (H) \) when the preference for capital \( (\beta) \) increases. That is, current consumption declines when the preference for capital \( (\beta) \) increases and the investment increases through the increase of retained earnings.

The second effect is the “risk effect” of the preference for capital \( (\beta) \), , which is the second term in the equation (17). The role of this effect hinges upon the sign of the product, \((1-\epsilon)(1-\gamma)\). If the sign of this product is negative, the risk effect makes the certainty effect stronger. However, if the sign of this product is positive, the risk effect may overpower the certainty effect. Therefore, it is possible for the preference for capital to decrease the growth rate of investment.

In other words, it normally would be expected that an increase in the preference for capital would cause the growth rate of investment to increase. However, the analysis suggests that under uncertainty it might actually decline the growth rate of investment. This paradoxical result may be explored by two empirical probable cases; \( \gamma > 1 \) and \( \epsilon < 1 \) and \( \gamma > 1 \) and \( \epsilon > 1 \).

Figure 1 illustrates the first case; \( \gamma > 1 \) and \( \epsilon < 1 \). Since \( \gamma > 1 \), the preference for capital increases effective risk aversion. Since \( \epsilon < 1 \), the growth rate of investment increases and uncertainty also increases the growth rate of investment. Therefore, in this case, the risk effect reinforces the time preference effect. The growth rate of investment is always larger under uncertainty and increases with the preference for capital (Figure 1).

Figure 2 depicts the second case; \( \gamma > 1 \) and \( \epsilon > 1 \). The growth rate of investment with uncertainty is below the one without uncertainty in this case. However, there are three possible circumstances. If \( \epsilon \rho < (1-\gamma)(1-\epsilon)\alpha^2 \sigma^2 \), the growth rate of investment decreases when the preference for capital increases (Figure 2a). If \((1-\gamma)(1-\epsilon)\alpha^2 \sigma^2 < \epsilon \rho < (1-\gamma)(1-\epsilon)\sigma^2 \), the growth rate of investment reaches the maximum and then decreases (Figure 2b). If \( \epsilon \rho > (1-\gamma)(1-\epsilon)\sigma^2 \), the growth rate of investment increases when the preference for capital increases (Figure 2c).
CONCLUDING REMARKS

In this note, we examine the investment behavior of a profit maximizing firm with the preference for capital under uncertainty. We found that the direction of the growth rate of investment depends on the magnitude of the intertemporal elasticity of substitution when uncertainty increases and the degree of risk aversion and the preference for capital affect the scale of the effect of uncertainty. Furthermore, the preference for capital may reduce the growth rate of investment through being the “certainty effect” overpowered by the “risk effect.”

In order to explain the dynamic investment behavior, the theoretical models require a number of assumptions for simplicity. Therefore, in future research, numerical analysis with real data will be interesting to verify the results of our model.

REFERENCES

Figure 1: $\varepsilon < 1, \gamma > 1$

\[ \sigma^2 > 0 \]

\[ \sigma^2 = 0 \]

\[ 0 \quad 1 - \alpha \beta \]

\[ g_t^* \]

\[ a. \ \varepsilon \rho < (1 - \gamma)(1 - \varepsilon)\alpha^2 \frac{\sigma^2}{\gamma} \]

\[ g_t^* \sigma^2 = 0 \]

\[ \sigma^2 > 0 \]

\[ 0 \quad 1 - \alpha \beta \]
Figure 2: z > 1, γ > 1, δ > 0