Advertising Expenditures Under Uncertainty:
A Duality Approach

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ABSTRACT

The purpose of this paper is to suggest an alternative theoretical model to analyzing firm advertising under uncertainty. With a duality approach, two probabilities as functions of advertising are defined for the dollar value of the buyer’s bundle of products and the time spent shopping. A possible equivalence in the expected value of these two entities is possible. This equivalence or lack thereof can affect the optimum level of advertising and result in decision-making implications that are not present in the literature. These implications are then explored.

Keywords: Advertising, Uncertainty, Bundle, Time Spent, Equivalence

INTRODUCTION

The advertising and related economic literature is vast (see, for surveys, Vakratsas and Amber 1999; Bagwell 2005; and Kim, et al. 2014), and with the advent of on-line shopping the advertising literature has been expanded to include entertainment and enjoyment aspects of shopping along with the traditional information and persuasive aspects (see, for a discussion, Childers, et al., 2001). Another recent aspect of the advertising literature is the recognition that buyers usually (or at least often) buy bundles of products and spend various amounts of time shopping in the store (see, on bundles, Harlam, et al. 1995; and for the time spent shopping see, Barli, et al. 2012; and Park, Lyer, and Smith 1989). In addition to the above aspects, there is the overriding condition of the uncertainty of the effect of advertising on buyers’ response in the form of purchases (see, for example, Nguyen1985; and Aykac, et al. 1989).

It is our purpose in this paper to propose a structure that integrates the above aspects of advertising into a theoretical model that has uncertainty, bundling, and time spent shopping as its key characteristics. Our design makes the expected dollar value of the typical buyer’s bundle of goods equal to the expected dollar value of the time the buyer spends in the store. Under uncertainty, these are known only probabilistically. Advertising has a dual effect and simultaneously determines the probability of the value of the bundle (say $\bar{P}(A)$) and the probability of the value of time spent in the store (say $\bar{P}(A)$).

As a result of the dual or simultaneous effects of advertising, there is a possibility that the equivalence between the expected value of the bundle and the expected value of time spent in the store, under uncertainty, may not hold. Our argument is that this possible equivalence (or lack, thereof) can affect the advertising optimization process and has implications for the type or form of advertising pursued by the management of the firm. Our main contribution to the advertising literature is the implications of the duality effect and equivalence for the firm’s advertising decision-making process.

To summarize briefly, our basic position is that firms are not particularly interested in the specific product a consumer buys, but in the bundle of products purchased over a given time period. For example, we suspect that Wal-Mart is interested in the total dollar expenditure of the whole bundle of goods the
The purpose of advertising is to increase the value of the bundle by getting the buyer to widen the bundle (in terms of more items like food, clothing, house wares, jewelry, toys, etc.) and to deepen the bundle (in terms of more quantity of each item). However, under uncertainty, the effect of advertising on the value of the bundle is not known with precision. Advertisers may then add other outcomes such as time spent shopping into their optimization. Differences in the probability functions associated with these two outcomes then have implications for advertising decisions. For the researcher in particular, looking at the wrong outcome can produce misleading predictions. For example, advertising may appear to be less than optimal when looking at the expected value of the bundle, but optimal when looking at the time spent in store. Advertising of various types will be used to increase the expected dollar value of the bundle and time spent shopping.

In what follows, in the next section the basic elements of the alternative approach are given and uncertainty is introduced into the approach. In the subsections, the direct and indirect approaches with corresponding probabilities under uncertainty are shown. Then, the next section contains the advertising optimization procedure. The next section considers the simultaneous dual effect of advertising on the probabilities associated with the bundle and time spent. The final section contains a summary and conclusions.

**AN ALTERNATIVE APPROACH**

For what follows, we define the total expenditure \( EX \) by the typical consumer on a given bundle using vector notation as \( EX(A) = p1X1 + p2X2 + \ldots + pnXn = pX(A) \), where \( A \) is advertising and the \( p_i \)'s are the given product prices. The width (given by \( n \)) and depth (given by the quantity-value of the \( X \)'s) of the bundle are the outcomes of the consumer's response to the amount and type of a given \( A \).

We also define an equivalent expression that reflects the time a buyer spends in the store shopping for the bundle. The time period will be affected by the depth and width of the bundle. The time-period expression is given by \( T(A) = t1X1 + t2X2 + \ldots + mXn = TX(A) \) in vector notation, where the \( t_i \)'s are the time-prices. Let \( r \) be the average money spent per unit of time (say, in dollars per minute). Then, under equivalence, \( EX(A) = rTX(A) \). In effect, the more time the typical buyer spends in the store, the more will be spent on the bundle.

Under uncertainty, however, equivalency may not hold, because of the way advertising affects the probabilities associated with the bundle and time spent in the store. From the firm's perspective, the buyer's choice solution \( EX(A) \) is not known with certainty, nor is the buyer's \( TX(A) \) time spent shopping known with certainty. In consideration of our duality argument, each of these uncertainties has its own corresponding probability function, each of which is dependent on advertising expenditures, \( A \). With these equivalent (possibly) expressions in mind, there are two ways to approach the advertising optimization problem, direct and indirect. In what follows, we consider both ways.

**The Direct Approach**

With the direct approach, the firm estimates the \( EX(A) \)'s for any given \( A \). The firm may produce an estimated frequency distribution of the typical buyer's \( EX(A) \)'s for different amounts of \( A \). For simplicity of model design, we assume that the firm estimates only two levels of expenditure on the bundle for any \( A \), a high level, \( EX_{\text{high}} \), with a probability of \( P(A) \) and a low level, \( EX_{\text{low}} \), with a probability of \( (1 - P(A)) \). The expected revenue for the optimization problem is given by \( ER(A) = P(A)EX_{\text{high}} + (1 - P(A))EX_{\text{low}} \). In
other words, we take the estimates of possible outcomes (high and low) as given with the corresponding probabilities.

**The Indirect Approach**

With the indirect approach, going back to the buyer’s time expression, we assume two $TX(A)$ estimates, $TX(A)_{\text{high}}$ and $TX(A)_{\text{low}}$ that correspond to the two probabilities, $\tilde{P}(A)$ and $(1 - \tilde{P}(A))$. Similar to before, the expected buyer’s time spend shopping is given by $ETX(A) = \tilde{P}(A)TX(A)_{\text{high}} + (1 - \tilde{P}(A))TX(A)_{\text{low}}$. The firm can make the two time estimates by clocking the entry and exit of buyers to the store, using suitable electronic devices.

To summarize briefly, we recognize two sources of uncertainty, time spent and money spent by the buyer. Both corresponding probabilities are jointly affected by $A$. But, as we will show, the effects need not be the same and the difference in the effects will have an impact on the optimization procedure.

**The Optimization Procedure**

The alternative approach under uncertainty focuses on how the firm chooses its optimal level of advertising, $A^*$, given its two probability functions, $P(A)$ and $\tilde{P}(A)$. This is the optimization problem. As indicated earlier, we consider two equivalent approaches to this problem. In the direct approach, as shown before, the firm’s expected revenue from the two possible bundles (high and low) is given by $ER(A) = P(A)EX_{\text{high}} + (1 - P(A))EX_{\text{low}}$. The firm’s total cost is given by $C(A, X) = C(X) + aA$, where for simplicity we assume that $C(X) = 0$. Also, with “$a$” we ignore any possible economies of scale or of scope. Expected profit is given by, $EP(A) = ER(A) - aA$. As before, we focus on the typical buyer, taken as given the actual number of buyers in the store at any given period of time. To simplify the optimization problem, we take as given the high/low estimates as indicated earlier. The firm’s optimal $A^*$ will then satisfy the first-order condition, $\partial EP(A)/\partial A = a/(EX_{\text{high}} - EX_{\text{low}})$, a positive constant by design.

Due to the equivalence principle we are proposing, the optimization problem for determining the optimal $A^*$ by the indirect approach will have a similar first-order condition given by $\partial \tilde{P}(A)/\partial A = a/r(ETX_{\text{high}} - ETX_{\text{low}})$, based on expected profit now given by $EP(A) = r[ \tilde{P}(A)TX(A)_{\text{high}} + (1 - \tilde{P}(A))TX(A)_{\text{low}} ] - aA$, where $r$ is the average time-price as given earlier.

**SIMULTANEITY PROBLEM**

As indicated earlier, spending on $A$ affects both probabilities $P(A)$ and $\tilde{P}(A)$ jointly. If equivalency holds, then the expected profit from the bundle and the time period will be the same. In other words, spending on $A$ has the same dual effect (like feeding the lamb produces both wool and mutton). With equivalency, the firm will be indifferent as to the amount or type of advertisement produced, one that increases the depth of the bundle versus one that increases the time spent in the store.

But, if $P$ and $\tilde{P}$ react differently to an increase in $A$, the firm will have a preference as to the amount and type of advertisement to produce. For example, say $P(A)$ rises faster than $\tilde{P}(A)$ as they both approach the probability limit of 1 (for illustration purposes, let $P(A) = aA^b$, where $0 < a < 1$ and $0 < b < 1$, say. 63 and 10 respectively and similarly for $\tilde{P}(A)$ but with $\tilde{b} = .05$). Then, the firm will expect more profit from the depth of the bundle (given its width, $n$) than from the time period. We can hypothesize that such a relative profit gain will affect the amount and type of advertisement, so the focus of $A$ will be on the depth of the bundle (by providing, for example, larger carts for more quantities) rather than on the time period spent in the store, per se.
On the other hand, the reverse may be the case, where $\bar{P}(A)$ rises faster than $P(A)$ as $A$ increases. In this situation, the firm will want to keep buyers in the store longer by various means (music, product demonstrations, product sampling, and the like).

Of course, if spending on $A$ equally affects both $P$ and $\bar{P}$, then the equivalency argument holds and the firm is indifferent as to the type of $A$ and reaches the same optimum quantity of advertising by either the direct or indirect approach. The problem with indifference is that the firm’s advertising manager can face a decision dilemma and indeterminacy. How such a dilemma is resolved in theory and in practice is beyond the scope of this paper, suffice it to say that more information may be needed in order to resolve the dilemma. We leave this open for future discussions.

SUMMARY AND CONCLUSIONS

An advertising model was designed based on the typical buyer’s depth and width of the bundle of products purchased. Also recognized was the time spent in the store purchasing the bundle. Uncertainty was introduced into the model, where managers may consider outcomes not directly tied to sales, and where two probabilities, one for the bundle and one for the time period, were defined. A duality was recognized in that both probabilities were dependent on advertising spending, $A$ (in other words, $A$ produces a joint probability). This duality or simultaneity could result in the equivalence between the expected dollar value of the bundle and the expected dollar value of the buyer’s time spent in the store. With such an equivalence, a decision problem may occur. If equivalence does not exist, then the firm will under optimization select the best (in terms of expected profit) type of advertisement program to follow, one that favors either the depth of the bundle or the length of time spent in the store.

Failure to recognize this duality may result in an incomplete understanding of advertising decisions. For example, a firm may over-spend on advertising relative to the expected value of the bundle but spend appropriately based on its estimate of the probability function and value of the time spent shopping. Therefore, these and other outcomes must be jointly considered to predict optimal advertising levels and type.

Our equivalence approach adds to the literature on advertising. The multi-role nature of advertising makes it a very complex phenomenon to fully understand. We hope that our equivalence approach has contributed something to a wider understanding of this complex phenomenon.

REFERENCES


