The Comparison of Grey Production Forecasting Models Based on Different Error Criteria

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ABSTRACT

Forecasting the production of companies or industries is important to entrepreneurs and governments and grey prediction model has been one of the popular forecasting techniques. Traditional grey model is derived from the criteria of least squares method, which may suffer from outliers in series and is not robust. In this paper, a grey model with minimizing the sum of absolute errors criteria is proposed and the parameters are estimated by the linear programming method. The proposed model could reduce the effect of outliers, which means that it is more robust than that obtained by least square criterion. Simulations are studied and the results demonstrate the efficiency of the approach. Real example is also presented in this research to show its application in practice.
Keywords: *Production Forecasting, Grey Model, Minimizing Sum of Absolute Errors Criteria, Outlier, Robustness.*

INTRODUCTION

Forecasting the production of companies or industries is important to entrepreneurs and governments (Wu, 1995). Marketing and production managers often need to forecast sales, earnings, costs, production levels, inventories, purchases, capital requirements, etc. Generally, Production forecasting techniques are classified as qualitative, involving primarily judgment, and quantitative, involving primarily historical data and mathematical models such as trend extrapolation, exponential smoothing, curvilinear fitting, regression analysis, input-output analysis, and econometrics model. Each method has different degrees of accommodation to forecast term, and each one has its own range of application and limitations. However, historical observations of production of companies or industries are sometimes influenced by interruptive events, such as strikes, outbreaks of war, sudden political or economic crises, unexpected heat or cold waves, or even unnoticed errors of typing and recording. The consequences of these interruptive events create spurious observations that are inconsistent with the rest of the series. Such observations are usually referred to as outliers and may lead to the quantitative production forecasting model invalid (Chen and Liu, 1993).

With rapid development in the study of forecasting techniques, several comparative methodologies have been evolved in the past decades. One special technique, Grey Model (GM) (Deng, 1982; Deng, 1989a; Deng, 1989b) has also been applied successfully in forecasting the behavior of system or series. Chang, Lai and Yu (2005) propose a variable P value rolling grey forecasting model for Taiwan semiconductor industry production; Lai (2002) and Lai (2003) applied GM group model to forecast non-stationary seasonal and nonlinear time series; Li, Yamaguchi and Nagai (2007) combine grey model and Markov chain to predict the number of Chinese international airlines. Other applications see Chang (2007), Fan (2002), Wang, and Hung, (2003), Xu and Wen, (1997). There are several advantages employing grey model to time series, such as easy calculation and few observations (data) needed. Grey model employs a first order psudo differential equation to characterize a system, and there are two parameters need to be estimated while model construction. These two parameters are obtained by least square criterion. However, as we know, the least square criterion may suffer from outliers in series and is not robust, so the grey model based on least square criterion is not robust either. In order to correct the deviations, a grey model with minimizing the sum of absolute errors criteria is proposed in this research and the parameters are estimated by the linear programming approach. Thus the parameters can be easily obtained by employing linear programming package such as EXCEL. The proposed model could reduce the effect of outliers, which means that it is more robust than that obtained by least square criterion. Simulations are studied and the results demonstrate the efficiency of the approach. Real example is also presented in this research to show its application in practice.

The organization of this paper is as follow. In section 2, the brief review of traditional grey model for production forecasting is made and the modified grey model with minimizing the sum of absolute errors criterion is proposed. In section 3, simulations are carried out to verify and compare the efficiency of the proposed model. A real world example is also employed to show the application in practice. Finally, some concluding remarks are made in section 4.

ESTABLISHMENT OF THE MATHEMATICAL MODEL

As described in previous section, Grey Model has also been applied successfully in forecasting the production of companies, demands of industries, etc. Here, a brief review of the most popular grey forecasting model, named GM(1,1) model, is made below.

Review of GM(1,1)

In this section, the GM(1,1) model is briefly reviewed. One may consult Deng (1982), Deng (1989a), Deng (1989b) for more details.

Given the data sequence $\{x^{(0)}(t), 1 \le t \le n\}$. Assume $x^{(0)}(t)$ is positive for all t. Then the pseudo difference equation of GM(1,1) is formed as

$$x^{(0)}(t) + a \cdot z^{(1)}(t) = b \tag{2.1}$$

where

$$z^{(1)}(t) = \frac{1}{2}(x^{(1)}(t) + x^{(1)}(t-1)) \tag{2.2}$$

The least square solution of (2.1) can be expressed as

$$\begin{bmatrix} a \\ b \end{bmatrix} = (B^T B)^{-1} B^T y \tag{2.3}$$

where

$$B = \begin{bmatrix} -z^{(1)}(2) & 1 \\ -z^{(1)}(3) & 1 \\ \vdots & \\ -z^{(1)}(n) & 1 \end{bmatrix} \tag{2.4}$$

and

$$y = \begin{bmatrix} x^{(0)}(2) \\ x^{(0)}(3) \\ \vdots \\ x^{(0)}(n) \end{bmatrix} \tag{2.5}$$

The sequence

$$\begin{aligned} x^{(1)} &= \{x^{(1)}(1), x^{(1)}(2), \ldots, x^{(1)}(n)\} \\ &= \left\{ \sum_{m=1}^{1} x^{(0)}(m), \sum_{m=1}^{2} x^{(0)}(m), \ldots, \sum_{m=1}^{n} x^{(0)}(m) \right\} \end{aligned} \tag{2.6}$$

is the accumulating generation operation (AGO) sequence. Accumulating generation exponential is an important scientific view in grey system theory (Deng, 1989a; Deng, 1989b; Fan, 2002) and is the basis of grey modeling also. The forecast of $\hat{x}^{(0)}(t)$ is then obtained by

$$\hat{x}^{(0)}(t) = \hat{x}^{(1)}(t) - \hat{x}^{(1)}(t-1) \tag{2.7}$$

where

$$\hat{x}^{(1)}(t) = (x^{(0)}(1) - \frac{b}{a})e^{-a\cdot(t-1)} + \frac{b}{a} \tag{2.8}$$

for $2 \le t \le n$ and $x^{(0)}(1) = x^{(1)}(1) = \hat{x}^{(1)}(1)$.

As we know, the least square criterion may suffer from outliers in series and is not robust, so the grey model based on least square criterion (as shown in equation (2.3)) is not robust either. In order to correct the deviations, a grey model with minimizing the sum of absolute errors criteria is proposed in the next sub-section.

The GM(1,1) Forecasting Model with Minimizing Sum of Absolute Deviation Criterion

The parameters of equation (2.1) are estimated by least square method and $\hat{x}^{(0)}(t)$ are obtained by (2.6) and (2.7). It is nature that one wishes to make the error between $\hat{x}^{(0)}(t)$ and $x^{(0)}(t)$ as small as possible. That is

$$\min\ C(a,b) = \min\ \sum_{t=1}^{n} |\varepsilon(t)| \tag{2.8}$$

where

$$\varepsilon(t) = x^{(0)}(t) - \hat{x}^{(0)}(t) \tag{2.9}$$

By Teng (1999) to eliminate the absolute in (3.1), define

$$\begin{cases} P(t) = \dfrac{|\varepsilon(t)| - \varepsilon(t)}{2}, i = 1,2,..,n. \\ Q(t) = \dfrac{|\varepsilon(t)| + \varepsilon(t)}{2}, i = 1,2,..,n. \end{cases} \tag{2.10}$$

Then $P(t) \ge 0$ and $Q(t) \ge 0$, for all $i = 1,2,..,n.$ Furthermore,

$$|\varepsilon(t)| = Q(t) + P(t), i = 1,2,..,n. \tag{2.11}$$

and

$$\varepsilon(t) = Q(t) - P(t), i = 1,2,..,n. \tag{2.12}$$

According (3.4) and (3.5), (3.1) and (3.2) can be replaced respectively by

$$\min C(a,b) = \min \sum_{t=1}^{n} \left(P(t) + Q(t)\right) \tag{2.13}$$

where

$$x^{(0)}(t) = \hat{x}^{(0)}(t) - P(t) + Q(T),\ \ i = 1,2,..,n. \tag{2.14}$$

Then the solution of a and b can be expressed as solving the linear programming problem below:

$$\min\ \sum_{t=1}^{n} \left(P(t) + Q(t)\right)$$

$$s.t.\quad \begin{cases} x^{(0)}(t) = \hat{x}^{(0)}(t) - P(t) + Q(t), i = 1,2,..,n. \\ P(t) \ge 0, i = 1,2,..,n. \\ Q(t) \ge 0, i = 1,2,..,n. \end{cases} \tag{2.15}$$

The parameters can be easily obtained by employing linear programming package such as EXCEL.

SIMULATION RESULTS AND APPLICATION

Simulation Settings

In order to investigate the practical implications of the models described above, an empirical study is carried out. For convenient, series with 5 observations are considered. Suppose the simulated data are generated from the following forms:

$$x_{ORI}^{(0)}(t) = 0.5 \cdot t + 1 + e(t)\,, t = 1,2,..,5 \tag{3.1}$$

where $e(t)$ denotes noises generated from normal distribution randomly with mean 0 and standard deviation 0.15. Note that function (3.1) is a linear function. To verify the proposed criteria is more robust than least square criteria, an inconsistent observation at $t = 3$ is made by the following forms in sequence:

$$x_{NEW}^{(0)}(t) = \begin{cases} 0.5 \cdot t + 1 + e(t), t = 1,2,4,5 \\ 0.5 \cdot t + 3 + e(t), t = 3 \end{cases} \tag{3.2}$$

with observation $x_{NEW}^{(0)}(3)$ is drift.

For each function, 50 sequences with five observations are generated. To compare the forecasting performance in general, the Mean absolute percentage error defined below is used:

$$MAPE = \left(\frac{1}{5} \sum_{t=1}^{5} \left| \frac{x_{NEW}^{(0)}(t) - \hat{x}_{NEW}^{(0)}(t)}{x_{NEW}^{(0)}(t)} \right| \right) \cdot 100\% \tag{3.3}$$

Furthermore, the absolute percentage error

$$APE = \left| x_{NEW}^{(0)}(3) - \hat{x}_{NEW}^{(0)}(3) \Big/ x_{NEW}^{(0)}(3) \right| \cdot 100\% \tag{3.4}$$

is also defined to measure the performance of $\hat{x}_i^{(0)}(4)$.

Simulation Results

Fig.1 shows the simulation results of (3.1) and (3.2). One may find that the MAPE values of grey model with least square error criterion is larger then those obtained with minimizing sum of absolute error criterion, which means the well fitting of the grey model with minimizing sum of absolute error criterion even the drift observations in the sequences as described in (3.2). The simulation results indicate the grey forecasting model with minimizing sum of absolute error criterion could reduce the effect of outliers, which means that it is more robust than that obtained by least square criterion.

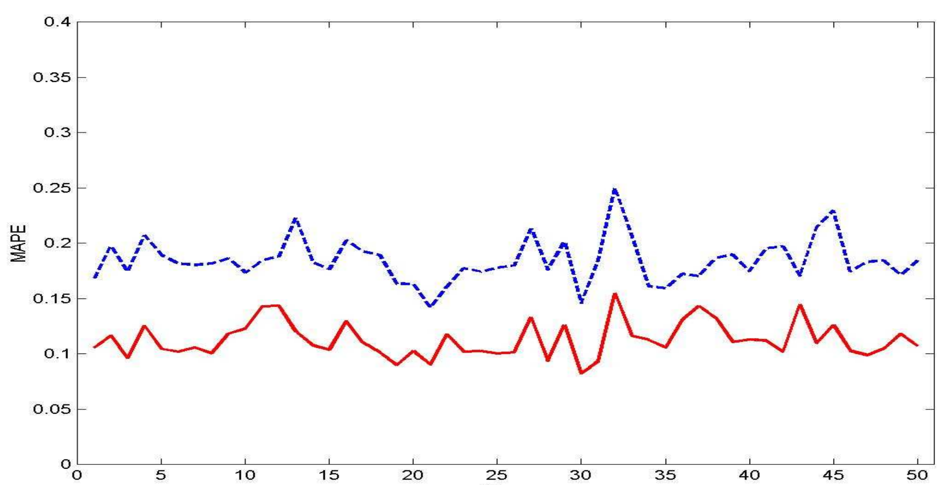

Figure 1: The MAPE Comparison of Grey Model with Least Square Error Criterion (dash line) and Grey Model Minimizing Sum of Absolute Deviation Criterion (solid line)

Furthermore, one may interest in the APE in Fig. 2. Definition of APE in (3.4) is to observe the forecasts of the drift points in sequences. Note that the drift points are made in order to simulate the situations of real case. Thus the larger APE means the forecasts far away from the drift points, which also shows the forecasts is less influence by drift points or outliers. Contrary to Fig.1, one may find that the APE values of grey model with minimizing sum of absolute error criterion are larger then those obtained by least square error criterion. This also show the grey forecasting model with minimizing sum of absolute error criterion could reduce the effect of outliers and is more robust than that obtained by least square criterion.

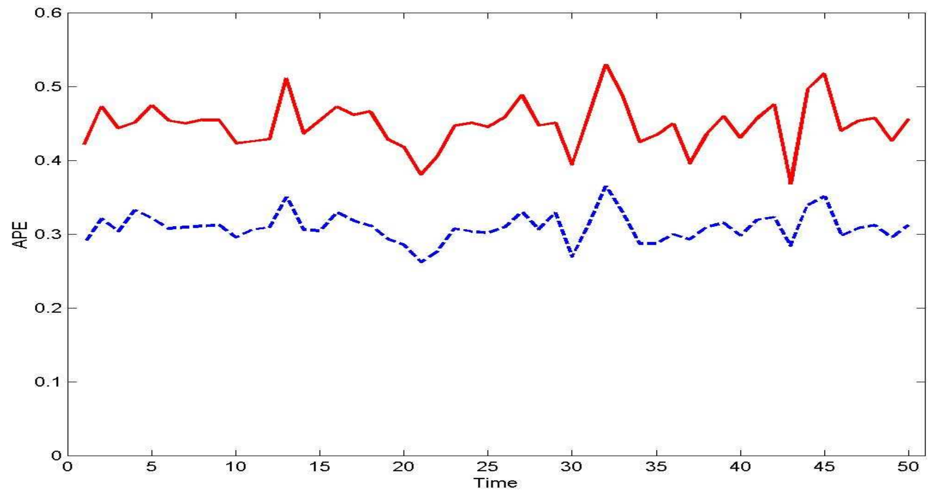

Figure 2: The APE Comparison of Grey Model with Least Square Error Criterion (dash line) and Grey Model Minimizing Sum of Absolute Deviation Criterion (solid line)

Application

Here we employ the example of U.S. quarterly series of beer production between 1975 and 1982, which was studied by Wei (1990). Note that in that example, the original observation $x_{ORI}^{(0)}(12) = 36.54$ was replaced by a new observation $x_{NEW}^{(0)}(12) = 56.54$, which could be due to a typing error. The observation $x_{NEW}^{(0)}(12)$ is then identified to be an outlier. Table 1 shows the original data.

Table 1: Quarterly U.S. Beer Production (Millions of Barrels)

Year	Quarter I	Quarter II	Quarter III	Quarter IV	
1975	36.14	44.60	44.15	35.72	
1976	36.19	44.63	46.95	36.90	$\boxed{56.54}$
1977	39.66	49.72	44.49	$\boxed{36.54}$	
1978	41.44	49.07	48.98	39.59	
1979	44.29	50.09	48.42	41.39	
1980	46.11	53.44	53.00	42.52	
1981	44.61	55.18	52.24	41.66	
1982	47.84	54.27	52.31	42.03	

Source: Wei (1990)

Suppose we are interesting in forecasting the production of the fourth quarter, then the observations of the fourth quarter each year are selected and re-index (Lai, 2003) as

$$\{x_{ORI}^{(0)}(4), x_{ORI}^{(0)}(8), x_{NEW}^{(0)}(12), \ldots, x_{ORI}^{(0)}(32)\} \overset{RE-INDEX}{=} \{x_{NEW}^{(0)}(1), x_{NEW}^{(0)}(2), \ldots, x_{NEW}^{(0)}(8)\} = x^{(0)}.$$

In order to compare grey production forecasting models based on different error criteria, the models are established by using the in-sample data from 1975 until 1980. The in-sample data are used to estimate the parameters of grey production forecasting models based on different error criteria. After the parameters are obtained, the series from the 1981 until 1982 are reserved as a post-sample, which are used to evaluate and compare the two step ahead forecasting abilities of both models.

Table 2 shows the results of in-sample and post-sample APE and MAPE respectively. The in-sample MAPE of grey forecasting model based on least square error criterion is 0.112, and the smaller in-sample MAPE of grey forecasting model based on minimizing sum of absolute error criterion is 0.0674. Especially, the APE forecasts at $t = 3$ by grey forecasting model based on minimizing sum of absolute error criterion is larger, which means that the forecasting value 38.54 is closer to the real value 36.54.

The post-sample forecasts of the 4-th quarter productions of 1981 and 1982 are also display in table2. The post-sample forecasts MAPE with different criteria 0.0088 and 0.061 respectively. Table 2 also indicates the grey production forecasting model based on minimizing sum of absolute error criterion could reduce the effect of outliers and is more robust than that obtained by least square criterion while applying in practice.

Table 2: Comparison of 4-th Quarter U.S. Beer Production Forecasts

			Grey Model based on least square error criterion		Grey Model based on minimizing sum of absolute error criterion	
	$x^{(0)}_{ORI}$	$x^{(0)}_{NEW}$	Forecasts	APE	Forecasts	APE
1	35.72	35.72	35.72		35.72	
2	36.90	36.90	44.14	0.1962	37.15	0.0067
3	36.54	56.54	43.76	0.2260	38.54	0.3184
4	39.59	39.59	43.38	0.0959	39.90	0.0078
5	41.39	41.39	43.01	0.0392	41.23	0.0040
6	42.52	42.52	42.64	0.0029	42.52	0.0000
	MAPE			0.1120		0.0674
7	41.66	41.66	42.28	0.0148	43.78	0.0510
8	42.03	42.03	41.91	0.0028	45.01	0.0710
	MAPE			0.0088		0.0610

CONCLUSION

Marketing and production managers often need to forecast sales, earnings, costs, and production levels before they make decision. Success in market competition can then be achieved with application of forecast method to service decision-making and consideration of some preventive measures when accuracy cannot be ensured in the forecast. A proper quantitative method is necessary for production forecasting task. However, it is more difficult to forecasting when historical observations of production of companies or industries are sometimes influenced by outliers.

In this paper, grey production forecasting models with different errors criteria are compared. Traditional grey production forecasting model is derived from the criteria of least squares method, which may suffer from outliers in series and is not robust. In this paper, a grey model with minimizing the sum of absolute errors criterion is carried out and the parameters are estimated by the linear programming method. A simulation involves 50 sequences with five observations are conducted to compare the forecasting performance between grey forecasting model with least squares criterion and model with minimizing the sum of absolute errors criterion. Simulation results indicate the grey forecasting model with minimizing the sum of absolute errors criterion could reduce the effect of outliers and is more robust then that obtained by least square criterion. U.S. quarterly beer productions between 1975 and 1982 are employed in this research to demonstrate the validation of grey production forecasting model with minimizing the sum of absolute errors criterion. The simulation and practice results show the grey production forecasting model with minimizing the sum of absolute errors criterion is a competitive and competent quantitative method for the production forecasting.

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