Performance of Risk Measures in China's Stock Markets

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ABSTRACT

This article examines the efficiency of five risk measures in the framework of portfolio optimization for the stocks of four China's stock markets and investigates which risk measure has the best performance in making asset allocation decisions. The data used are the historical monthly stock returns from 1998 to 2002. Although the downside risk measures are thought to be consistent with investors' actual perception of risk, our finding shows that the traditional Mean-Variance model is much more efficient and has the best performance in forming global minimum risk portfolios among all five risk measures.

INTRODUCTION

The Mean-Variance (MV) theory of portfolio selection, which proposed by Markowitz (1952), offers investors the opportunity to construct their portfolios on the basis of a risk/return tradeoff. Although this theory marked an epoch in the history of financial investment, the MV portfolio optimization was not widely used in the past. One of the reasons is that the error in the inputs (expected returns and covariances) could result in wrong asset allocation decision. Recently, several trends suggest that professional investors and academics are conscious of the importance of risk estimation in asset allocation decisions. In fact, the errors in the risk inputs can be important, particularly in estimating equilibrium returns or in index replication, where expected returns are not required as an input.

Compared to estimating the risk, the difficulty of estimating expected returns and sensitivity of portfolio performances to the error in estimating expected returns implies that the most improvement that can actually be made on MV optimization lies in risk estimation. Some studies point out the importance of this issue, for example, Chopra and Ziemba (1993) demonstrate that asset allocation decisions are most sensitive to errors in expected returns rather than errors in variances and covariances. Eun and Resnick (1988) show that portfolio strategies, which adjust for estimation risk, lead to superior portfolio performance.

Although adopting risk estimation as input might be an attractive way to solve the portfolio selection problem, there is still much debate over the appropriate definition of risk. Some academics argue that the theoretical assumptions of MV approach to support variance as a risk measure are somewhat restrictive and variance is not consistent with investors' actual perception of risk (Bawa, 1975; Fishburn, 1977; Harlow, 1991). They advocate adopting downside risk as an alternative risk measure. Some studies also show that empirical optimizations based on downside risk measures are more efficient than mean-variance measures in the sense that the selected portfolios have less downside exposure than those determined using variance (Harlow, 1991; Cheng, 2001).

The purpose of this study is to find out the appropriate risk measure for asset allocation decisions in China's stock markets and to investigate whether the performance of stock portfolio in China's stock markets using downside risk measures superior that using MV measure.

This article is structured in the following way. In section two we describe the data and the methods we use. The results of the empirical investigation would be reported in section three. Section four contains the conclusion of this article.

DATA AND MEHTODOLOGY

The data used consist of the monthly returns from the four stock markets in China, namely Shanghai A-Share stocks and B-Share stocks as well as Shenzhen A-Share stocks and B-Share stocks, and spans from January 1998 to December 2002. The data are from Shanghai Stock Exchange and Shenzhen Stock Exchange.

Since our purpose is to investigate which kind of risk measure has the best performance in making asset allocation decisions, we examine this issue in the context of a bootstrap simulation study where the portfolio allocations aim to minimize portfolio risk.

For each stock market the simulation would be run five hundred times for each risk measure. In every simulation, 25 stocks of each stock market would be randomly selected. Using the first twelve monthly return observations of the selected stocks, their expected sample risk for the 13^{th} month would be calculated and the portfolio optimization would be implemented. After the optimal portfolio is found, the expected and the realized portfolio return of the 13^{th} month would be calculated. For the 14^{th} month, the second to the 13^{th} monthly return observations would be used, and so on. Since we have 60 monthly returns for each stock market, there are 48 pairs of expected and realized portfolio returns in each simulation. The means of the expected and realized portfolio returns would then be tested as to see whether they are the same. To ensure that there exist realized returns for the analysis, the newly listed and the delisted stock during the sample period would be excluded. Table 1 represents the numbers of the stock used in these four markets.

Table 1. Numbers of the stock used for analysis

Shanghai A-Shares	Shanghai B-Shares	Shenzhen A-Shares	Shenzhen B-Shares
356	36	328	47

The null hypothesis is

$$H_0 : \mu_1 = \mu_2 ,$$

and the test statistic is

$$t_{n-1} = \frac{\mu_1 - \mu_2}{\sqrt{\dfrac{S_1^2 + S_2^2}{n}}} ,$$

where μ_1 is the expected sample mean of 48 portfolios and μ_2 stands for their realized sample mean. S_1^2 and S_2^2 represents the variance of the expected and the realized portfolio returns, respectively. n is the number of observation and is set to be 48 in our cases.

The risk measures used in this study are the Mean-Lower Semi-Absolute Deviation, Mean-Lower Semi-Variance, Mean-Below Target Risk, Mean-CVaR models, and traditional Mean-Variance model.

Mean-Lower Semi-Absolute Deviation Model (M-LSAD)

The lower semi-absolute deviation is defined as follows:

$$\sigma_j = E\left[\left|R_j - \overline{R}_j\right|\right],$$

where $\left|R_j - \overline{R}_j\right| = \max\{0, -(R_j - \overline{R}_j)\}$, R_j is the rate of return of the stock j and $\overline{R}_j$ is its average. This model is a convex function of w and most portfolios on the efficient frontier generated by mean-lower semi-absolute deviation model are consistent with the principle of maximization of expected utility (Ogryczak and Ruszczynski, 1999).

Since the lower semi-absolute deviation could be written as

$$E\left[\left|R_j - \overline{R}_j\right|\right] = \sum_{t=1}^{T} p_t \left|\sum_{j=1}^{n} (r_{jt} - r_j) w_j\right| ,$$

the optimization problem could be therefore formulated as follows:

$$Min! \sum_{t=1}^{T} p_t \left|\sum_{j=1}^{n} (r_{jt} - r_j) w_j\right| \tag{1}$$

$$s.t.\ w_j \ge 0, \quad j = 1, \cdots, n,$$

or

$$Min! \sum_{t=1}^{T} p_t z_t$$

$$s.t.\ z_t \ge -\sum_{j=1}^{n} (r_{jt} - r_j) w_j , \quad t = 1, \cdots, T$$

$$z_t \ge 0, \quad t = 1, \cdots, T$$

$$w_j \ge 0, \quad j = 1, \cdots, n,$$

where r_{jt} represents the realized return of stock j at time t and r_j is its average. p_t is the probability that

r_{jt} is attained and is set to be $1/T$. w_j is the proportion of the fund to be allocated to stock j .

Since an optimal solution ($w_1^*, w_2^*, \cdots, w_n^*, z_1^*, z_2^*, \cdots, z_n^*$) of this problem satisfies the relation

$$z_t^* = \max\left\{0, -\sum_{j=1}^{n}\left(r_{jt} - r_j\right)w_j^*\right\} = \left|\sum_{j=1}^{n}\left(r_{jt} - r_j\right)w_j^*\right|,$$

therefore ($w_1^*, w_2^*, \cdots, w_n^*$) is an optimal solution of (1).

Mean-Lower Semi-Variance Model (M-LSV)

The most well known downside risk model is the lower semi-variance of Markowitz (1959):

$$\sigma_j^2 = E\left[\left|R_j - \overline{R}_j\right|^2\right].$$

Lower semi-standard deviation is the square root of the lower semi-variance. It is well known that this model and MV model are consistent with the principle of maximization of expected utility, if R_j follows normal distribution. Also, optimal portfolios of MV, M-LSAD and M-LSV are usually similar when the underlying assets follow symmetric distribution (Ogryczak and Ruszczynski, 1999).

The optimization problem could be formulated as follows:

$$Min! \sum_{t=1}^{T} p_t \left|\sum_{j=1}^{n}\left(r_{jt} - r_j\right)w_j\right|^2 \qquad (2)$$

$$s.t.\ w_j \geq 0, \quad j = 1, \cdots, n$$

or

$$Min! \sum_{t=1}^{T} p_t z_t^{\ 2}$$

$$s.t.\ z_t \geq -\sum_{j=1}^{n}\left(r_{jt} - r_j\right)w_j, \quad t = 1, \cdots, T$$

$$z_t \geq 0, \quad t = 1, \cdots, T$$

$$w_j \geq 0, \quad j = 1, \cdots, n$$

Mean-Below Target Risk Model (M-BT)

The target rate of return τ is defined as the average return of the market index. The below target risk of order k is defined as follows:

$$\sigma_j = E\left[\left|R_j - \tau\right|^k\right]^{1/k}.$$

This risk measure is also consistent with the principle of maximization of expected utility and is a convex function of w for all k (Fishburn, 1977). From the computational point of view, the case of k =1 and 2 are usually used in order to reduce the problem to linear and quadratic programming problem, respectively. In this study k is set to be unity.

In case of k =1, the below target risk is represented as follows:

$$\sigma_j = E\left[\left|R_j - \tau\right|\right] = \sum_{t=1}^{T} p_t \sum_{j=1}^{n}\left|r_{jt} - \tau\right|.$$

Therefore, the optimization problem of M-BT model could be formulated as follows:

$$
\begin{aligned}
&Min!\ \sum_{t=1}^{T} p_t z_t \\
&s.t.\quad z_t \geq -\sum_{j=1}^{n} r_{jt} w_j + \tau,\quad t = 1,\cdots,T \\
&\qquad z_t \geq 0,\quad t = 1,\cdots,T \\
&\qquad w_j \geq 0,\quad j = 1,\cdots,n
\end{aligned} \tag{3}
$$

Mean-CVaR Model (M-CVaR)

It is well known that the VaR model is not valid when return exhibits a longer tail distribution. Also, it is not a convex function of w so that it is very difficult to solve minimization problems. An alternative measure of risk of value-at-risk (VaR) model is the conditional value-at-risk (CVaR) model, which is sometimes called expected shortfall and maintains advantages of VaR, yet free from computational disadvantages of VaR model.

Let $L(w)$ be the loss function associated with portfolio w. Then the conditional value-at-risk is defined as follows:

$$CVaR_\beta(w) = \frac{1}{1-\beta} E\left[L(w) \middle| L(w) \geq VaR_\beta(w)\right],$$

where $VaR_\beta(w)$ is defined as the smallest number α_β such that $P_r\{L(w) \geq \alpha_\beta\} = 1-\beta$. In this study α and β is set to be 0.05 and 0.95, respectively.

Since the theorem of Rockafellar and Uryasev (2002) shows that CVaR can be minimized by using convex minimization algorithms, the linear programming problem of minimizing $CVaR_\beta(w)$ could be formulated as follows:

$$
\begin{aligned}
&Min!\ \alpha + \sum_{t=1}^{T} p_t z_t / (1-\beta) \\
&s.t.\quad z_t \geq -\sum_{j=1}^{n} r_{jt} w_j + \tau - \alpha,\quad t = 1,\cdots,T \\
&\qquad z_t \geq 0,\quad t = 1,\cdots,T \\
&\qquad w_j \geq 0,\quad j = 1,\cdots,n,
\end{aligned} \tag{4}
$$

where τ is some constant and is set to be the average return of the stock market index.

Mean-Variance Model (MV)

The traditional MV model is employed here, where the task is to find the global minimum variance portfolio. The optimization problem is formulated as follows:

$$
\begin{aligned}
&Min!\ \sum_{i=1}^{n} \sum_{j=1}^{n} w_i w_j \sigma_{ij} \\
&s.t.\ \overline{R}_p = \sum_{j=1}^{n} w_j \overline{R}_j \\
&\qquad \overline{R}_j = \frac{1}{T} \sum_{t=1}^{T} R_t \\
&\qquad \sigma_j^2 = \frac{1}{T-1} \sum_{t=1}^{T} \left(R_t - \overline{R}_j\right)^2 \\
&\qquad w_j \geq 0,\quad t = 1,\cdots,T,\quad j = 1,\cdots,n.
\end{aligned} \tag{5}
$$

EMPIRICAL RESULTS

Table 2 shows the results of simulations for each risk measure in all four markets. As can be seen in Table 2, MV model has the smallest number of times that the null hypothesis is rejected, namely the smallest number of times that the expected portfolio returns and the average realized portfolio returns are statistically different. It implies that the traditional risk measure is much more efficient and has the best performance in forming minimum risk portfolios in all these risk measures.

Table 2. The number of times of simulations that the null hypothesis is rejected for each risk measure in all four markets

	Test Statistic	M-LSAD	M-LSV	M-BT	M-VaR	MV
Shanghai A-Share	Insiginficant	482	494	465	465	500
	Siginficant	18	6	35	35	0
Shanghai B-Share	Insiginficant	500	500	409	409	500
	Siginficant	0	0	91	91	0
Shenzhen A-Share	Insiginficant	483	489	456	448	499
	Siginficant	17	11	44	52	1
Shenzhen B-Share	Insiginficant	500	500	494	494	500
	Siginficant	0	0	6	6	0

CONCLUSION

This article examines the efficiency of five risk measures in the framework of portfolio optimization for the stocks of four China's stock markets and investigates which risk measure has the best performance in making asset allocation decisions. The data used are the historical monthly stock returns from 1998 to 2002. In order to investigate which kind of risk measure has the best performance in making asset allocation decisions, we examine this issue in the context of a bootstrap simulation study where the portfolio allocations aim to minimize portfolio risk.

Although the downside risk measures are thought to be consistent with investors' actual perception of risk, our finding shows that the traditional MV model is much more efficient and has the best performance in forming global minimum risk portfolios among all these five risk measures.

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