Modified Controllable Lead Time Model with Stochastic Demand and Batch Shipment Policy

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ABSTRACT

This study investigates the single-vendor single-buyer integrated production inventory models with controllable lead time. The lead time demand is stochastic and shortage during the lead time is permitted. The buyer orders a lot of size Q and the vendor manufactures mQ with a finite production rate P at one set-up but ships in quantity Q to the buyer over m times. Most researchers do not take this assumption into account while determining the crashing cost. In this study, a modified model is proposed under the practical fact that the production is continuous during a production cycle in which the crashing set-up cost is charged only once. Numerical examples are presented to illustrate the procedures and results of the proposed algorithm. The modified model is shown to provide lower total costs and shorter lead time.
Keywords: *Controllable lead time; Lead time crashing costs; Batch shipments policy.*

INTRDUCTION

Time-based competition focuses on the reduction of overall system response time and inventory lead time reduction has been one of favorite topics for both researchers and practitioners (Pan and Hsiao, 2005). Liao and Shyu (1993) first presented a stochastic inventory model with lead time being the variable. Ben-Daya and Raouf (1994) modified Liao and Shyu model (1993) by including both lead time and order quantity as decision variables. Ouyang et al. (1996) extended Ben-Daya and Raouf's (1994) model by allowing shortages and treated the stockout to be a mixture of backorders and lost sales. Ouyang and Wu (1998) developed a minimax distribution free procedure for mixed inventory model with variable lead time. Pan and Hsiao (2001) modified Ouyang et al. model (1996) by considering back-order discount. Pan et al. (2002) assumed the crash cost is a function of both the order quantity and the reduced lead time, and then established inventory models with fixed and variable lead time crash cost. Pan and Yang (2002) improved Goyal model (1988) by considering lead time as a controllable factor. Ouyang et al. (2004) extended Pan and Yang model (2002) by assuming lead time demand is stochastic and shortage during lead time is permitted. Pan and Hsiao (2005) investigated an integrated inventory system in which shortage is allowed and both lead time and backordering are negotiable. Chang et al. (2006) proposed integrated vendor-buyer cooperative inventory models with controllable lead time and ordering cost reduction.

In a single-vendor single-buyer integrated inventory model, adopting a batch shipment policy can significantly reduce total cost. Szendrovits (1975) presented a model for reducing both manufacturing cycle time and total cost by using equal sized batches during all stages for a multi-echelon model where transportation costs are considered sunk costs. Subsequently, Goyal (1976) proposed a search procedure for determining economic production quantity and optimal batch numbers for the Szendrovits model (1975). While assuming equal batch size among different stages and permitting variation in batch number, Goyal (1978), Szendrovits and Drezner (1980) proposed estimating economic batch quantity and optimal batch number at every stage with constant lot size. Furthermore, Goyal (1977) developed a method for optimizing production quantity in a two-stage production system involving unequal batch sizes in an increasing or decreasing geometric series. Moreover, Goyal and Szendrovits (1986) presented a constant lot size model with both equal and variable sized batch shipments between production stages, and proposed a heuristic solution. Lu (1995) devised a heuristic method for solving the one-vendor multi-buyer problem. Furthermore, Goyal (1995) incorporated a policy in which size of successive shipments from manufacturer to customer within a production cycle increases by a factor equal to the ratio of production to demand. Hill (1997) presented a more general class of policy by proposing that the shipment size factor should be smaller than the ratio of production to demand. Hill (1999) derived the structure of the globally-optimal policy in which the final size of the equal-sized shipments may not equal the largest batch size among the unequal-sized shipments. Additionally, Hoque and Goyal (2000) incorporated the capacity constraints of the transport equipment into the vendor-buyer integrated model. Zhou and Wang (2007) developed a more general production-inventory model. The problem was solved using the nonlinear programming subroutine of Matlab 5.3. The unit holding cost of the vendor is lower than that of the buyer, their model outperforms the previous models in reducing total cost. However, the complexity of the Zhou and Wang model (2007) exceeds that of the Hill model (1999).

This study addresses the production and inventory problem involving single-vendor single-buyer integrated system. A stochastic model is constructed under conditions of equal sized batch shipments and controllable lead time. During the lead time, shortages are allowed and fully backordered. Firstly, a integrated system model with controllable lead time and batch shipment is constructed. Moreover, a modified model is established using reasonable concept: allocated the crashing set-up cost to each batch. An iterative procedures algorithm is developed for determining the optimal solutions. Furthermore, numerical examples are presented to illustrate the procedures and results of the proposed algorithm.

NOTATIONS AND ASSUMPTIONS

The notations used in this study listed as follows:

Q Order quantity of the buyer (decision variable).

R Reorder point of the buyer (decision variable).

L Length of lead time for the buyer (decision variable).

m The number of batches in which the product is delivered from the vendor to the buyer in one production cycle, a positive integer (decision variable).

D Average demand per unit time on the buyer.

P Production rate on the vendor.

A Buyer’s ordering cost per order.

S Vendor’s set-up cost per set-up.

C_v Unit production cost paid by the vendor.

C_b Unit purchase cost paid by the buyer, $C_b > C_v$.

r_v Vendor’s holding cost rate per unit time.

r_b Buyer’s holding cost rate per unit time.

π Unit backorder cost for the buyer.

The assumptions of the model in this study are listed as follows:

(a) There is single-vendor and single-buyer for a single product in this model.

(b) The product demand X follows a normal distribution with finite mean D and variance σ^2.

(c) The buyer orders a lot of size Q and the vendor manufactures mQ with a finite production rate $P(P>D)$ at one set-up but ship in quantity Q to the buyer over m times. The vendor incurs a set-up cost S for each production run and the buyer incurs an ordering cost A for each order of quantity Q.

(d) Inventory is continuously reviewed. The buyer places the order when the on hand inventory reaches the reorder point R.

(e) The reorder point $R = DL + k\sigma_L$, where k is the safety factor and $\sigma_L^2 = \sigma^2 L$.

(f) Shortages are allowed and fully backordered.

(g) The lead time L consists of n mutually independent components. The ith component has a normal duration b_i, minimum duration a_i, and crashing cost per unit time c_i. For convenience, we rearrange c_i such that $c_1 < c_2 < \ldots < c_n$.

The components of lead time are crashed one at a time starting from the first component because it has the minimum unit crashing cost, and then the second component, and so on.

MODEL CONSTRUCTION

Ouyang et al. (2004) proposed integrated vendor-buyer cooperative models with controllable lead time under the long-term strategic partnerships between buyer and vendor are well established. Lead time can be decomposed into n mutually independent components, each of which has a different crashing cost for reduce lead time. The ith component has maximum duration b_i and minimum duration a_i. Let L_i be the length of lead time component i crashed to its minimum duration, $i = 1, 2, \dots, n$, then L_i can be expressed as:

$$L_i = \sum_{j=1}^{n} b_j - \sum_{j=1}^{i} (b_j - a_j) \tag{1}$$

The lead time crashing cost $C(L)$ for a given $L \in [L_i, L_{i-1}]$ is given by:

$$C(L) = c_i(L_{i-1} - L) + \sum_{j=1}^{i-1} c_j(b_j - a_j) \tag{2}$$

The joint total expected cost per unit time for the vendor and buyer is given by

$$JTEC(Q,R,L,m) = \frac{D}{Q}[G(m) + \pi E(X-R)^+ + C(L)] + \frac{QH(m)}{2} + r_b C_b (R - DL), \tag{3}$$

where $G(m) = A + \frac{S}{m}$ and $H(m) = r_b C_b + r_v C_v \left[m\left(1 - \frac{D}{P}\right) - 1 + \frac{2D}{P} \right]$.

The vendor sets the facilities up with set-up cost S to produce the item in the quantity of mQ, and the purchaser would receive it in m lots, with which each having a quantity of Q. The vendor manufactures the product in the quantity of mQ with a finite production rate P at one set-up.

Set-up time could be crashed with extra charges (Liao and Shyu, 1991). Suppose the eth component of lead time is the set-up time which can be crashed with crashing set-up cost per unit time c_e. Since the production is continuous during a replenishment cycle, the crashing set-up cost is charged only once. In other words, we do not need the crashing set-up cost to charge after the first lot is produced.

In Equation (3) first term $G(m) = A + \frac{S}{m}$, $\frac{S}{m}$ means that the set-up cost is allocated to each lot. Therefore, the crashing set-up cost should also be allocated to each lot. And the lead time crashing cost $C(L)$ for a given $L \in [L_i, L_{i-1}]$ should be modified as follows:

$$C_{new}(L) = c_i(L_{i-1} - L) + \frac{\alpha c_e}{m}(b_e - a_e) + \sum_{j=1 \, and \, j \neq e}^{i-1} c_j (b_j - a_j), \tag{4}$$

where $\alpha = 1$ if the eth component of lead time is crashed and $\alpha = 0$ if the eth component of lead time is not crashed.

In addition, the joint total expected cost per unit time should be modified as follows:

$$JTEC_{new}(Q,R,L,m) = \frac{D}{Q}[G(m) + \pi E(X-R)^+ + C_{new}(L)] + \frac{QH(m)}{2} + r_b C_b (R - DL), \tag{5}$$

Since $C_{new}(L) \le C(L)$ for any given L, we have

$$JTEC_{new}(Q,R,L,m) \le JTEC(Q,R,L,m) \tag{6}$$

The product demand X follows a normal distribution with finite mean D and variance σ^2. Therefore, the lead time demand X has a normal distribution with mean DL and variance $\sigma_L^2 = \sigma^2 L$. The reorder point $R = DL + k\sigma_L$, where k is the safety factor. Then the expected shortage at the end of the batch cycle is given by $\sigma\sqrt{L}\psi(k)$, where $\psi(k)=\phi(k)-k[1-\Phi(k)]$, and ϕ, Φ are the standard normal distribution and cumulative distribution function, respectively. The joint total expected cost per unit time can be rewritten as follows:

$$JTEC_{Nnew}(Q,k,L,m)=\frac{D}{Q}[G(m)+\pi\sigma\sqrt{L}\psi(k)+C_{new}(L)] + \frac{QH(m)}{2}+r_bC_bk\sigma\sqrt{L} \ . \tag{7}$$

The optimal order quantity and safety factor should be modified as follows:

$$Q=\left\{\frac{2D[G(m)+\pi\sigma\sqrt{L}\psi(k)+C_{new}(L)]}{H(m)}\right\}^{1/2} \tag{8}$$

and $$\Phi(k)=1-\frac{r_bC_bQ}{\pi D} \tag{9}$$

THE ITERATIVE PROCEDURE ALGORITHM

The following iterative procedures can be used to find the optimal order quantity, the number of batches, the reorder point, and the joint total expected cost per unit time.

Step 1. Set m = 1 and $JTEC_{Nnew}* = \infty$.
Step 2. Rearrange crashing costs c_j such that $c_1 \le c_2 \le \ldots \le c_e/m \le \ldots. \le c_n$.
Step 3. For each L_i perform (i) to (v), i = 0, 1, 2, … , n.
(i) Set k_{i1} =0.
(ii) Substitute $\psi(k_{i1})$ into Eq. (8) to evaluate Q_{i1}.
(iii) Utilize Q_{i1} to determine $\Phi(k_{i2})$ from Eq. (9), then finds k_{i2} and $\psi(k_{i2})$.
(iv) Repeat (ii)-(iii) until no change occurs in the values of Q_i and k_i.
(v) Compute the corresponding $JTEC_{Nnew}(Q_i, k_i, L_i, m)$.
Step 4. Find $JTEC_{Nnew}(Q^*_m, k^*_m, L^*_m, m) = \min_{i=0,1,2,\ldots,n} JTEC_{Nnew}(Q_i, k_i, L_i, m)$.
Step 5. If $JTEC_{Nnew}(Q^*_m, k^*_m, L^*_m, m) \le JTEC_{Nnew}*$,
then set $JTEC_{Nnew}* = JTEC_{Nnew}(Q^*_m, k^*_m, L^*_m, m)$,
$Q_N* = Q^*_m$, $k_N* = k^*_m$, $L_N* = L^*_m$, $m_N* = m$, $m = m + 1$,
and go to *Step* 2.
Step 6. (Q_N*, k_N*, L_N*, m_N*) is the optimal solution and $JTEC_{Nnew}*$ is the optimal joint total expected cost per unit time.

NUMERICAL EXAMPLES

Example 1

Consider an inventory system with the data used in Ouyang et al. (2004): D = 600 units/year, A = \$200/order, C_b = \$100/unit, π = \$50/unit, σ = 7 units/week, P = 2000 units/year, S = \$1500/set-up, C_v = \$70/unit, r_b = 0.2, r_v =0.2 and lead time has three components with data shown in Table 1. Assume that the lead time demand follows a normal distribution and the 2nd component of lead time is the crashing set-up.

Table 1: Lead time data

Lead time component i	Normal duration b_i (days)	Minimum duration a_i (days)	Unit crashing cost c_i (\$/day)
1	20	6	0.4
2	20	6	1.2
3	16	9	5.0

Applying the Algorithm of Ouyang et al. yields the results as shown in Table 2. Use our modified Algorithm, the crash sequence is changed to be components 2, 1, 3 while $m \geq 4$. The results are shown in Table 3.

From Table 3, we obtain the optimal ordering quantity $Q^* = 143$ units, lead time $L^* = 28$ days, number of lots $m^* = 3$, safety factor $k^* = 1.305$ and the minimum joint total expected annual cost is $6612.0. The modified model is shown to provide lower total costs compared with those of Ouyang et al. models.

Table 2: Summary of the solution of Ouyang et al. model

m	L_m*(days)	k_m*	Q_m*	$JTEC_N(Q_m^*, R_m^*, L_m^*, m)$
1	28	0.84	299	$7466.7
2	28	1.14	189	$6760.0
3	28	1.31	144	$6660.4←
4	28	1.41	118	$6722.5

'←' denotes the minimum joint total expected annual cost.

Table 3: Summary of the solution of our model

m	L_m*(days)	k_m*	Q_m*	$JTEC_{Nnew}(Q_m^*, R_m^*, L_m^*, m)$
1	28	0.84	299	$7466.7
2	28	1.14	189	$6733.3
3	28	1.305	143	$6612.0←
4	28	1.418	117	$6657.9

'←' denotes the minimum joint total expected annual cost.

Example 2

Consider an inventory system with the data: $D = 1200$ units/year, $A = \$200$/order, $C_b = \$100$/unit, $\pi = \$50$/unit, $\sigma = 10$ units/week, $P = 2000$ units/year, $S = \$1500$/set-up, $C_v = \$70$/unit, $r_b = 0.2$, $r_v = 0.2$ and lead time has three components with data shown in Table 1. Assume that the lead time demand follows a normal distribution and the 3rd component of lead time is the crashing set-up.

Applying the Algorithm of Ouyang et al. yields the results as shown in Table 4. Use our modified Algorithm, the crash sequence is changed to be components 1, 3, 2 while $m \geq 5$. The results are shown in Table 5.

From Table 4, we have the optimal ordering quantity $Q^* = 182$ units, lead time $L^* = 28$ days, number of lots $m^* = 4$, safety factor $k^* = 1.55$ and the minimum joint total expected annual cost is $8844.3.

From Table 5, we obtain the optimal ordering quantity $Q^* = 161$ units, lead time $L^* = 21$ days, number of lots $m^* = 5$, safety factor $k^* = 1.61$ and the minimum joint total expected annual cost is $8739.5. The modified model is shown to provide lower total costs and shorter lead time compared with those of Ouyang et al. models.

Table 4: Summary of the solution of Ouyang et al. model

m	L_m*(days)	k_m*	Q_m*	$JTEC_N(Q_m^*, R_m^*, L_m^*, m)$
1	28	1.14	386	$11488.8
2	28	1.35	267	$9633.2
3	28	1.47	214	$9051.9
4	28	1.55	182	$8844.3←
5	42	1.615	159	$8853.3

'←' denotes the minimum joint total expected annual cost.

Table 5: Summary of the solution of our model

m	L_m*(days)	k_m*	Q_m*	$JTEC_{Nnew}(Q_m^*, R_m^*, L_m^*, m)$
1	28	1.14	386	$11488.8
2	21	1.34	269	$9614.5
3	21	1.46	215	$9015.0
4	21	1.54	183	$8795.9
5	21	1.61	161	$8739.5←
6	21	1.66	145	$8766.3

'←' denotes the minimum joint total expected annual cost.

CONCLUSION

This paper proposes a good method for set-up time improvement and can have impact in some cases. Although the total costs are not much lower than those of Ouyang et al.'s numerical examples, the lead time is reduced significantly from 28 days to 21 days in Example 2.

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